

Parallel Texture Structures with Cofactor Zeros in Lepton Sector

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Abstract

In this paper we investigate the parallel texture structures with cofactor zeros in the charged lepton and neutrino sectors. The textures can not be obtained from arbitrary leptonic matrices by making weak basis transformations, which therefore have physical meaning. The 15 parallel textures are grouped as 4 classes where each class has the same physical implications. It is founded that one of them is not phenomenological viable and another is equivalent to the texture zero structures extensively explored in previous literature. Thus we focus on the other two classes of parallel texture structures and study the their phenomenological implications. The constraints on the physical variables are obtained for each class, which are essential for the model selection and can be measured by future experiments. The model realization is illustrated in a radiated lepton mass model.

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I. INTRODUCTION

The discovery of neutrino oscillations has provided us with convincing evidences for massive neutrinos and leptonic flavor mixing with high degree of accuracy[1–3]. The recent measurement of large reactor mixing angle θ_{13} has not only open the door for us to explore the leptonic CP violation and the mass hierarchy in the future experiments, but also highlight the flavor puzzle of neutrino mass and mixing pattern which appears to be rather different from the distinct mass hierarchy and the small mixing angles shown by quarks. Although a full theory is still missing, several ideas have been proposed by reducing the number of free parameters of seesaw models[4] and introducing the specific structures into mass matrices to explain the observed leptonic mixing pattern. The models include texture zeros[5], hybrid textures[6, 7], zero trace[8], zero determinant[9], vanishing minors[10–12], two traceless submatrices[13], equal elements or cofactors[14], hybrid M_ν^{-1} textures[15]. Among these models, the textures with zero elements or zero minors are particularly interesting because of their connection to the flavor symmetries and the stable behavior of running renormalization group. The phenomenological analysis of neutrino mass matrices with texture zeros or cofactor zeros in flavor basis have been widely investigated in earlier literature[5, 10–12].

However, there is no priori requirement that the analysis must be done in flavor basis. The more general situation should be considered in the basis where both charged lepton mass matrix M_l and neutrino mass matrix M_ν are non-diagonal. In this spirit, the parallel *Ansätze* has been proposed where M_l and M_ν have the same structure (We denote it "parallel texture structure"). A popular parallel texture structure appears as the Fritzsch-like model[16] with texture zeros in mass matrix and is firstly applied to understand the quark mixing pattern. Subsequently the idea is generalized to the lepton sector[17, 18]. A systematic search on the parallel structures with texture zeros in lepton mass matrices are reported in Ref.[19]. It is shown that some sets of the texture zeros have no physical meaning by themselves, since they can be obtained by making suitable weak basis (WB) transformation from

arbitrary mass matrix and leaving the gauge currents invariant. The minimal non-trivial case is the four texture zeros model. Recently, a similar investigation is done in the context of parallel hybrid textures with one zero and two equal elements [20].

In this work, we study the parallel structures with two cofactor zeros in both M_ν and M_l . As shown in Ref. [11], the cofactor zeros in M_ν are generated by the type-I seesaw formula $M_\nu = -M^D M_R^{-1} M_D^T$ with texture zeros in M_D and M_R . The cofactor zeros in M_l , on the other hand, seems to be rather unusual because no flavor symmetry directly leads to the cofactor zeros of Dirac mass matrix M_l . However, we will show that if we adopt the recent viewpoint proposed by Ma[21] that the radiated lepton mass originate from the one-loop diagram, a seesaw-like formula is possible for charged lepton masses and the cofactor zeros in M_l can be realized. There exists $C_6^2 = 15$ logically possible patterns. Furthermore, we assume the mass matrices to be Hermitian and all neutrinos are massive, which indicates $\det M_\nu \neq 0$ and existence of M_ν^{-1} . Thus the mass textures M_ν with cofactor zeros are equivalent to the M_ν^{-1} with texture zeros. As the texture zero case [19], the 15 textures structures can be grouped into 4 classes with each class having the same physical implications. Among the 4 classes, we find that one of them is not viable phenomenologically and another class is equal to the matrices with texture zeros. Therefore we focus on the other two classes having not been studied before.

The paper is organized as follows. In Sec. II, we discuss the classification of textures and relate them to the experimental results. In Sec.III, we diagonalize the mass matrices, confront the numerical results with the experimental data and discuss their predictions. In Sec. IV, the realization of cofactor zeros in M_l is discussed. A summary is given in Sec. IV.

II. FORMALISM

A. Weak basis equivalent classes

We assume the neutrinos to be Majorana fermions. The most general WB transformations leaving gauge currents invariant is given by

$$M_l \rightarrow M'_l = W^\dagger M_l W_R \quad M_\nu \rightarrow M'_\nu = W^T M_\nu W \quad (1)$$

where W, W_R are 3×3 unitary matrices. Therefore the parallel texture with cofactor zeros located at different positions can be related by permutation matrix P as the WB transformation

$$M'_l = P^T M_l P \quad M'_\nu = P^T M_\nu P \quad (2)$$

The permutation matrix P changes the positions of cofactor zeros but preserves the parallel structures for both charged lepton and neutrino mass textures. It is noted that P belongs to the group of 6 permutations and are isomorphic to S_3 . Then the four cofactor zeros texture can be classified into 4 classes as following:

Class I:

$$\begin{pmatrix} \Delta & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} \Delta & \Delta & \times \\ \Delta & \times & \times \\ \times & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & \Delta & \times \\ \Delta & \Delta & \times \\ \times & \times & \times \end{pmatrix} \\ \begin{pmatrix} \times & \times & \times \\ \times & \Delta & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \Delta \end{pmatrix} \quad \begin{pmatrix} \times & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \Delta \end{pmatrix} \quad (3)$$

Class II:

$$\begin{pmatrix} \Delta & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & \Delta \\ \times & \Delta & \times \\ \Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & \Delta & \times \\ \Delta & \times & \times \\ \times & \times & \Delta \end{pmatrix} \quad (4)$$

Class III:

$$\begin{pmatrix} \Delta & \times & \times \\ \times & \Delta & \times \\ \times & \times & \times \end{pmatrix} \quad \begin{pmatrix} \Delta & \times & \times \\ \times & \times & \times \\ \times & \times & \Delta \end{pmatrix} \quad \begin{pmatrix} \times & \times & \times \\ \times & \Delta & \times \\ \times & \times & \Delta \end{pmatrix} \quad (5)$$

Class IV:

$$\begin{pmatrix} \times & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \times \end{pmatrix} \quad \begin{pmatrix} \times & \Delta & \times \\ \Delta & \times & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad \begin{pmatrix} \times & \times & \Delta \\ \times & \times & \Delta \\ \Delta & \Delta & \times \end{pmatrix} \quad (6)$$

where " Δ " at (i, j) position denotes the zero cofactor $C_{ij} = 0$ while " \times " stands for arbitrary element. Since $M_{l,\nu}$ with cofactor zeros is equivalent to $M_{l,\nu}^{-1}$ with zero elements, the classification given above is the same as the texture zero ones shown in Ref.[19] except for replacing " Δ " with "0". Like the texture zero cases, the class IV leads to the decoupling of a generation of lepton from mixing and thus not experimentally viable. On the other hand, one can easily check that the textures of class I correspond to the texture zero ones, which has already studied in previous literature[17–19]. As an example, for the first matrix of class I, we have

$$M_{l,\nu} = \begin{pmatrix} \Delta & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \times \end{pmatrix} \Rightarrow M_{l,\nu}^{-1} = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \Rightarrow M_{l,\nu} = \begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix} \quad (7)$$

Therefore only class II and class III have no trivial physical implications.

B. Some useful notations

As we have mentioned, we only need to investigate two mass matrices respectively belonging to representations of class II and class III. In this work, we choose

$$M_{l,\nu}^{II} = \begin{pmatrix} \Delta & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad M_{l,\nu}^{III} = \begin{pmatrix} \Delta & \times & \times \\ \times & \Delta & \times \\ \times & \times & \times \end{pmatrix} \quad (8)$$

The charged leptonic mass texture M_l is a complex Hermitian matrix and the Majorana neutrino mass texture M_ν is a complex symmetric matrix. They are diagonalized by unitary matrix V_l and V_ν

$$M_l = V_l M_l^D V_l^\dagger \quad M_\nu = V_\nu M_\nu^D V_\nu^T \quad (9)$$

where $M_l^D = \text{Diag}(m_e, m_\mu, m_\tau)$, $M_\nu^D = \text{Diag}(m_1, m_2, m_3)$. The Pontecorvo-Maki-Nakagawa-Sakata matrix[22] U_{PMNS} is given by

$$U_{PMNS} = V_l^\dagger V_\nu \quad (10)$$

and can be parameterized as

$$U_{PMNS} = U P_\nu = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix} \quad (11)$$

where the abbreviations $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ are used. The α and β in P_ν denote two Majorana CP-violating phases and δ in U denotes the Dirac CP-violating phase. In order to facilitate our calculation, it is better to start from M_l^{-1} rather than M_l . From (9), we get

$$M_l^{-1} = V_l (M_l^D)^{-1} V_l^\dagger \quad (12)$$

So the V_l can not only diagonalize the M_l but also M_l^{-1} . Furthermore, we treat the Hermitian matrix M_l^{-1} to be factorisable. i.e

$$M_l^{-1} = K_l (M_l^{-1})^r K_l^\dagger \quad (13)$$

where K_l is the unitary phase matrix and can be parameterized as $K_l = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$. The $(M_l^{-1})^r$ becomes a real symmetric matrix which can be diagonalized by real orthogonal matrix O_l . Then we have

$$V_l = K_l O_l \quad (14)$$

and

$$U_{PMNS} = O_l^T K_l^\dagger V_\nu \quad (15)$$

From (9), (10) and (15), the neutrino mass matrix M_ν is given by

$$M_\nu = K_l V P_\nu M_\nu^D P_\nu V^T K_l^\dagger \quad (16)$$

where $V \equiv O_l U$. From (16) The restriction of two cofactor zeros on M_ν

$$M_{\nu(pq)}M_{\nu(rs)} - M_{\nu(tu)}M_{\nu(vw)} = 0 \quad M_{\nu(p'q')}M_{\nu(r's')} - M_{\nu(t'u')}M_{\nu(v'w')} = 0 \quad (17)$$

induces two equations

$$m_1 m_2 K_3 e^{2i\alpha} + m_2 m_3 K_1 e^{2i(\alpha+\beta+\delta)} + m_3 m_1 K_2 e^{2i(\beta+\delta)} = 0 \quad (18)$$

$$m_1 m_2 L_3 e^{2i\alpha} + m_2 m_3 L_1 e^{2i(\alpha+\beta+\delta)} + m_3 m_1 L_2 e^{2i(\beta+\delta)} = 0 \quad (19)$$

where

$$K_i = (V_{pj}V_{qj}V_{rk}V_{sk} - V_{tj}V_{uj}V_{vk}V_{wk}) + (j \leftrightarrow k) \quad (20)$$

$$L_i = (V_{p'j}V_{q'j}V_{r'k}V_{s'k} - V_{t'j}V_{u'j}V_{v'k}V_{w'k}) + (j \leftrightarrow k) \quad (21)$$

with (i, j, k) a cyclic permutation of (1,2,3). After solving Eq.(18) and (19), we arrive at

$$\frac{m_1}{m_2} e^{-2i\alpha} = \frac{K_3 L_1 - K_1 L_3}{K_2 L_3 - K_3 L_2} \quad (22)$$

$$\frac{m_1}{m_3} e^{-2i\beta} = \frac{K_2 L_1 - K_1 L_2}{K_3 L_2 - K_2 L_3} e^{2i\delta} \quad (23)$$

With the help of Eq.(22) and (23), we obtain the magnitudes of mass ratios

$$\rho = \left| \frac{m_1}{m_3} e^{-2i\beta} \right| \quad (24)$$

$$\sigma = \left| \frac{m_1}{m_2} e^{-2i\alpha} \right| \quad (25)$$

as well as the two Majorana CP-violating phases

$$\alpha = -\frac{1}{2} \arg \left(\frac{K_3 L_1 - K_1 L_3}{K_2 L_3 - K_3 L_2} \right) \quad (26)$$

$$\beta = -\frac{1}{2} \arg \left(\frac{K_2 L_1 - K_1 L_2}{K_3 L_3 - K_2 L_3} e^{2i\delta} \right) \quad (27)$$

The results of Eq. (24),(25), (26) and (27) imply that the two mass ratios (ρ and σ) and two Majorana CP-violating phases (α and β) are fully determined in terms of

the real orthogonal matrix O_l and $U(\theta_{12}, \theta_{23}, \theta_{13}$ and δ). The neutrino mass ratios ρ and σ are related to the ratios of two neutrino mass-squared ratios obtained from the solar and atmosphere oscillation experiments as

$$R_\nu \equiv \frac{\delta m^2}{\Delta m^2} = \frac{2\rho^2(1 - \sigma^2)}{|2\sigma^2 - \rho^2 - \rho^2\sigma^2|} \quad (28)$$

and to the three neutrino mass as

$$m_2 = \sqrt{\frac{\delta m^2}{1 - \sigma^2}} \quad m_1 = \sigma m_2 \quad m_3 = \frac{m_1}{\rho} \quad (29)$$

where $\delta m^2 \equiv m_2^2 - m_1^2$ and $\Delta m^2 \equiv |m_3^2 - \frac{1}{2}(m_1^2 + m_2^2)|$. In the numerical analysis, we use the latest global-fit neutrino oscillation experimental data, at 3σ confidential level, which is listed in Ref.[24]

$$\begin{aligned} \sin^2 \theta_{12}/10^{-1} &= 3.08_{-0.49}^{+0.51} & \sin^2 \theta_{23}/10^{-1} &= 4.25_{-0.68}^{+2.16} & \sin^2 \theta_{13}/10^{-2} &= 2.34_{-0.57}^{+0.63} \\ \delta m^2/10^{-5} &= 7.54_{-0.55}^{+0.64} eV^2 & \Delta m^2/10^{-3} &= 2.44_{-0.22}^{+0.22} eV^2 \end{aligned} \quad (30)$$

for normal hierarchy (NH) and

$$\begin{aligned} \sin^2 \theta_{12}/10^{-1} &= 3.08_{-0.49}^{+0.51} & \sin^2 \theta_{23}/10^{-1} &= 4.25_{-0.74}^{+2.22} & \sin^2 \theta_{13}/10^{-2} &= 2.34_{-0.61}^{+0.61} \\ \delta m^2/10^{-5} &= 7.54_{-0.55}^{+0.64} eV^2 & \Delta m^2/10^{-3} &= 2.40_{-0.23}^{+0.21} eV^2 \end{aligned} \quad (31)$$

for inverted hierarchy(IH). There is no constraint on the Dirac CP-violating phase δ at 3σ level, however, the recent global fit tends to give $\delta \approx 1.40\pi$. In neutrino oscillation experiments, CP violation effect is usually reflected by the Jarlskog rephasing invariant quantity[23] defined as

$$J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta \quad (32)$$

The Majorana nature of neutrino can be determined if any signal of neutrinoless double decay is observed, implying the violation of leptonic number violation. The decay ratio is related to the effective of neutrino m_{ee} , which is written as

$$m_{ee} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}| \quad (33)$$

Although a 3σ result of $m_{ee} = (0.11 - 0.56)$ eV is reported by the Heidelberg-Moscow Collaboration[25], this result is criticized in Ref [26] and shall be checked by the forthcoming experiment. It is believed that that the next generation $0\nu\beta\beta$ experiments,

with the sensitivity of m_{ee} being up to 0.01 eV[27], will open the window to not only the absolute neutrino mass scale but also the Majorana-type CP violation. Besides the $0\nu\beta\beta$ experiments, a more severe constraint was set from the recent cosmology observation. Recently, an upper bound on the sum of neutrino mass $\sum m_i < 0.23$ eV is reported by Planck Collaboration[28] combined with the WMAP, high-resolution CMB and BAO experiments.

III. PHENOMENOLOGICAL IMPLICATIONS OF PARALLEL COFACTOR ZERO TEXTURES

A. Class II

In this section, we study the phenomenological implications of class II. The factorisable formation of inverse charged leptonic matrix $(M_l^{-1})^r$ are parameterized as

$$(M_l^{-1})_{II}^r = \begin{pmatrix} 0 & a & c \\ a & b & 0 \\ c & 0 & d \end{pmatrix} \quad (34)$$

and can be diagonalized by an orthogonal matrix O_l

$$O_l^T (M_l^{-1})_{II}^r O_l = \text{diag}(m_e^{-1}, -m_\mu^{-1}, m_\tau^{-1}) \quad (35)$$

where the coefficients a, c, d are real and positive; The m_e, m_μ and m_τ denote the mass eigenvalues of charged leptons for three generations. The minus sign in (35) has been introduced to facilitate the analytical calculation and has no physical meaning since the charged lepton is Dirac fermions. Using the invariant $\text{Tr}(M_l^{-1})^r$, $\text{Det}(M_l^{-1})^r$ and $\text{Tr}(M_l^{-1})^{r^2}$ the nonzero elements of $(M_l^{-1})^r$ can be expressed in terms of three mass eigenvalues m_e, m_μ, m_τ and d

$$a = \sqrt{-\frac{(m_e^{-1} - m_\mu^{-1} - d)(m_e^{-1} + m_\tau^{-1} - d)(-m_\mu^{-1} + m_\tau^{-1} - d)}{m_e^{-1} - m_\tau^{-1} + m_\tau^{-1-2d}}} \quad (36)$$

$$b = m_e^{-1} - m_\mu^{-1} + m_\tau^{-1} - d \quad (37)$$

$$c = \sqrt{\frac{(d - m_e^{-1})(d + m_\mu^{-1})(d - m_\tau^{-1})}{m_e^{-1} - m_\tau^{-1} + m_\tau^{-1}}} - 2d \quad (38)$$

where the parameter d is allowed in the range of $0 < d < m_\tau^{-1}$ and $m_e^{-1} - m_\tau^{-1} < d < m_e^{-1}$. Then the O_l can be easily constructed as

$$O_l = \begin{pmatrix} \frac{(b-m_e^{-1})(d-m_e^{-1})}{N_1} & \frac{(b+m_\mu^{-1})(d+m_\mu^{-1})}{N_2} & \frac{(b-m_\tau^{-1})(d-m_\tau^{-1})}{N_3} \\ -\frac{a(d-m_e^{-1})}{N_1} & -\frac{a(d+m_\mu^{-1})}{N_2} & -\frac{a(d-m_\tau^{-1})}{N_3} \\ -\frac{c(b-m_e^{-1})}{N_1} & -\frac{c(b+m_\mu^{-1})}{N_2} & -\frac{c(b-m_\tau^{-1})}{N_3} \end{pmatrix} \quad (39)$$

where the a, b and c in (39) is given in (36), (37) and (38); The N_1, N_2 and N_3 are the normalized coefficients given by

$$N_1^2 = (b - m_e^{-1})^2(d - m_e^{-1})^2 + a^2(d - m_e^{-1})^2 + c^2(b - m_\tau^{-1})^2 \quad (40)$$

$$N_2^2 = (b + m_\mu^{-1})^2(d + m_\mu^{-1})^2 + a^2(d + m_\mu^{-1})^2 + c^2(b + m_\mu^{-1})^2 \quad (41)$$

$$N_3^2 = (b - m_\tau^{-1})^2(d - m_\tau^{-1})^2 + a^2(d - m_\tau^{-1})^2 + c^2(b - m_\tau^{-1})^2 \quad (42)$$

Substitute the O_l we obtained into (39) to (24), (25), (26), (27) and (28), the ratio of mass squared difference can be expressed via eight parameters. i.e three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, one Dirac CP-violating phase δ , three charged lepton mass (m_e, m_μ, m_τ) and a parameter d . Here we choose the three charged leptonic masses at the electroweak scale ($\mu \simeq M_Z$) i.e [29]

$$m_e = 0.486570154 \text{ MeV} \quad m_\mu = 102.7181377 \text{ MeV} \quad m_\tau = 1746.17 \text{ MeV} \quad (43)$$

In the numerical analysis, We randomly vary the three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ in their 3σ range and parameter d in its proper range. Up to now, no bound was set on Dirac CP-violating phase δ at 3σ level, so we vary it randomly in the range of $[0, 2\pi]$. Using Eq. (28), the mass-squared difference ratio R_ν is determined. Then the input parameters is empirically acceptable when the R_ν falls inside the the 3σ range of experimental data, otherwise they are excluded. Finally, we get the value of neutrino mass and Majorana CP-violating phase α and β though Eq.(24), (25). Since we have already obtained the absolute neutrino mass $m_{1,2,3}$, the further constraint from cosmology should be considered. In this work, we set the upper bound on the sum of neutrino mass Σm_i less than 0.23 eV.

We present the numerical results of class II in Fig.1 for the NH and in Fig.2 for the IH. One can see from the figures that different mass spectra exhibit different correlations between physical variables. For the NH case, the Dirac CP-violating phase δ is highly restricted in the range of $60^\circ \sim 70^\circ$ and leads to the Jarlskog rephasing invariant $|J_{CP}| > 0.02$ which is promising to be detected in the future long baseline neutrino oscillation experiments. On the other hand, there exists a bound of $\theta_{23} > 48^\circ$. Although accepted at 3σ level, this result is phenomenologically ruled out at 2σ level since recent experiments tend to give $\theta_{23} < \pi/4$. We obtain the bound on the lightest neutrino mass M_1 , $0.025eV < m_1 < 0.075eV$ and the effective Majorana neutrino mass m_{ee} $0.04eV < m_{ee} < 0.10eV$ which reaches the accuracy of future neutrinoless double beta decay ($0\nu\beta\beta$) experiments. The correlation between α and β is also illustrated that the small range is allowed at 3σ level. For the IH case, the three mixing angles θ_{12} , θ_{23} , and θ_{13} are fully covered the 3σ range while the constrained Dirac CP-violated phase δ lies in the range of $70^\circ \sim 290^\circ$, leading to the $|J_{CP}| \sim (0) - (0.04)$. Interestingly, m_{ee} and the lightest neutrino mass m_3 exhibit a strong dependence on δ . Such correlations are essential for the model selection and could be tested by experiments. There also exists a bound of $0.005eV < m_{ee} < 0.095eV$ which could be in principle tested by future $0\nu\beta\beta$ experiments. The Majorana phase α is covered in the whole range of $-90^\circ \sim 90^\circ$ while β is constrained in the range of $-25^\circ \sim 25^\circ$.

B. Class III

Let's consider another class of textures which is phenomenologically interesting. In the factorisable case, the real matrix $(M_l^{-1})^r$ is parameterized as

$$(M_l^{-1})^r_{III} = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & d \end{pmatrix} \quad (44)$$

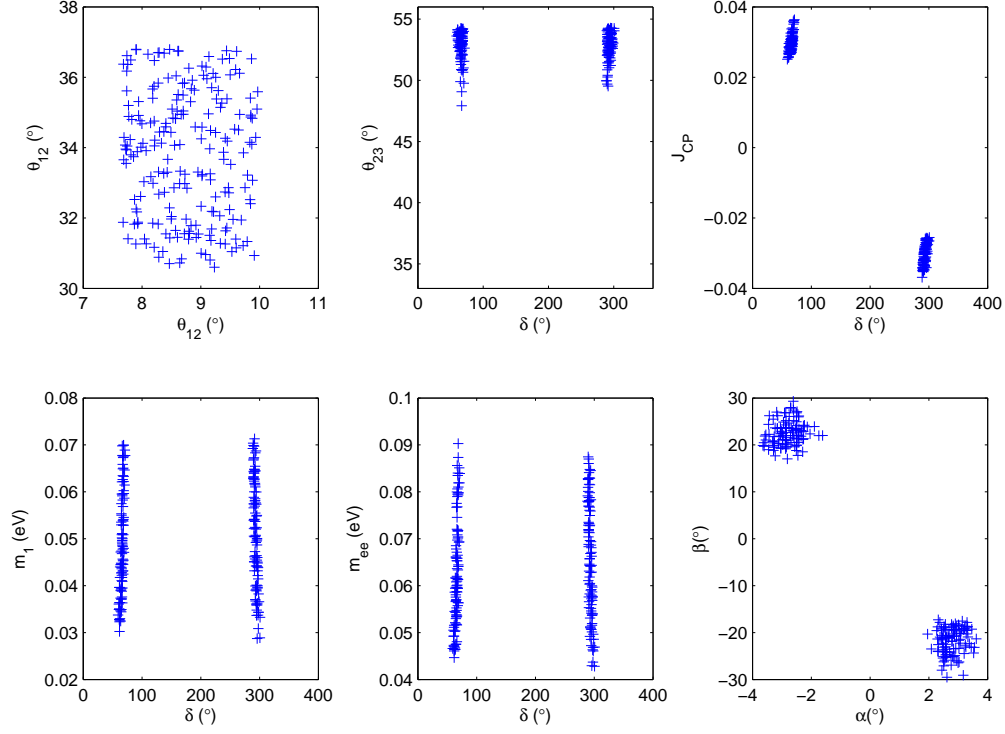


Figure 1: The correlation plots for class II(NH).

where a, b, c and d are real number. Without loss of generality, the parameter b, c are set to be positive. The matrix can be diagonalized by the orthogonal matrix O_l

$$O_l^T (M_l^{-1})^r_{III} O_l = \text{diag}(m_e^{-1}, -m_\mu^{-1}, m_\tau^{-1}) \quad (45)$$

Different from class II, we choose a as the free parameter since the trace of $(M_l^{-1})^r$ has already fixed d to be

$$d = m_e^{-1} - m_\mu^{-1} + m_\tau^{-1} \quad (46)$$

Using the invariant $\text{Det}(M_l^{-1})^r$ and $\text{Tr}(M_l^{-1})^{r^2}$, the parameters b, c can be expressed by three charged leptonic mass eigenvalues (m_e, m_μ, m_τ) and a

$$(b \pm c)^2 = -(-m_e^{-1}m_\mu^{-1} + m_e^{-1}m_\tau^{-1} - m_\mu^{-1}m_\tau^{-1}) - a^2 \pm \frac{a^2(m_e^{-1} - m_\mu^{-1} + m_\tau^{-1}) - m_e^{-1}m_\mu^{-1}m_\tau^{-1}}{a} \quad (47)$$

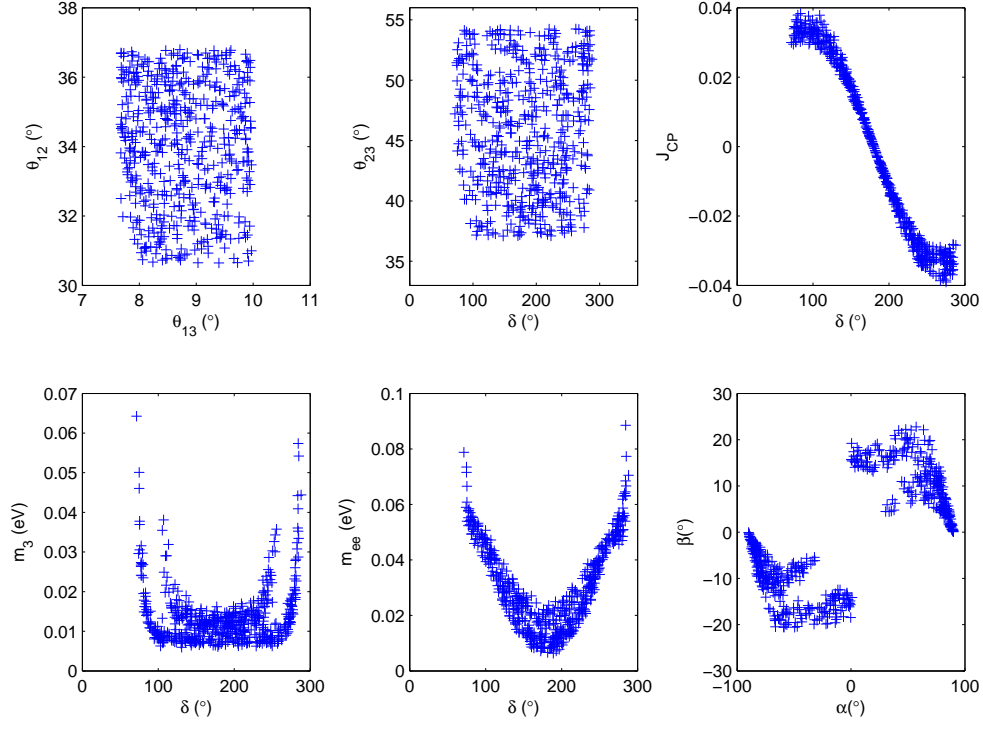


Figure 2: The correlation plots for class II(IH).

With the help of Eq. (46) and Eq. (47), one can construct the diagonalized matrix O_l to be

$$(M_l^{-1})_{III}^r = \begin{pmatrix} \frac{O(11)}{N_1} & \frac{O(12)}{N_2} & \frac{O(13)}{N_3} \\ \frac{O(21)}{N_1} & \frac{O(22)}{N_2} & \frac{O(23)}{N_3} \\ \frac{O(31)}{N_1} & \frac{O(32)}{N_2} & \frac{O(33)}{N_3} \end{pmatrix} \quad (48)$$

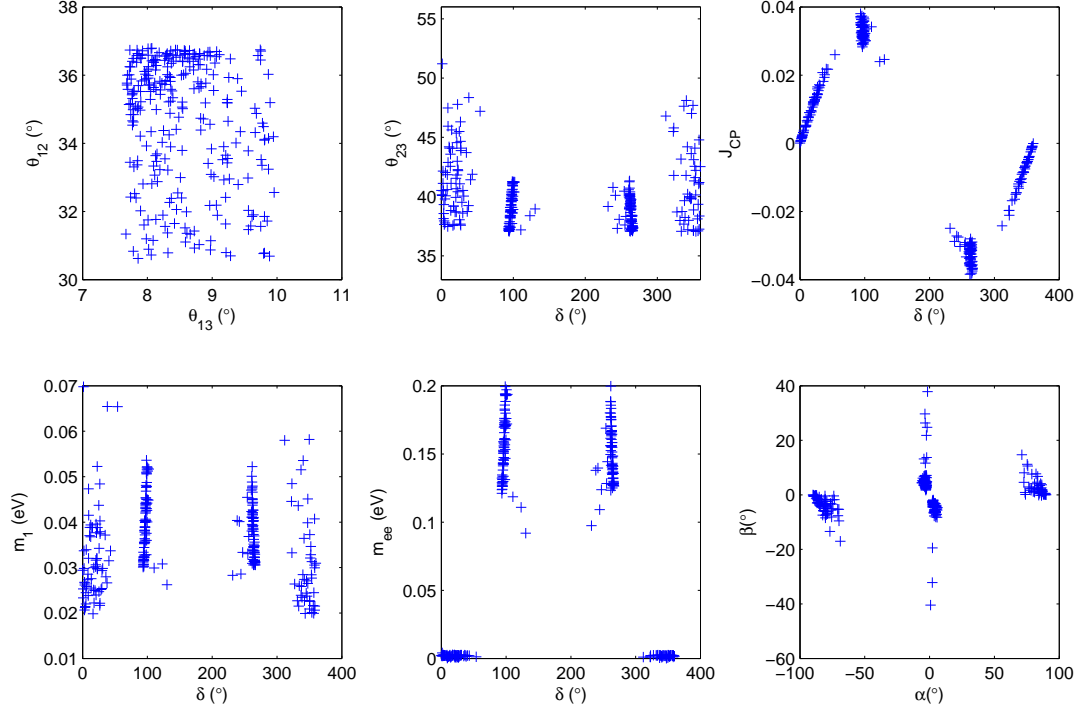


Figure 3: The correlation plots for class III(NH).

where

$$\begin{aligned}
O(11) &= am_e(bm_e + ca^{-1}) + bm_e(m_e^{-1}a^{-1} - m_e a) \\
O(12) &= -am_\mu(-bm_\mu + ca^{-1}) - bm_\mu(-m_\mu^{-1}a^{-1} + m_\mu a) \\
O(13) &= am_\tau(bm_\tau + ca^{-1}) + bm_\tau(m_\tau^{-1}a^{-1} - m_\tau a) \\
O(21) &= bm_e + ca^{-1} \\
O(22) &= -bm_\mu + ca^{-1} \\
O(23) &= bm_\tau + ca^{-1} \\
O(31) &= m_e^{-1}a^{-1} - m_e a \\
O(32) &= -m_\mu^{-1}a^{-1} + m_\mu a \\
O(33) &= m_\tau^{-1}a^{-1} - m_\tau a
\end{aligned} \tag{49}$$

and the normalized coefficients is given by

$$\begin{aligned} N_1^2 &= O(11)^2 + O(21)^2 + O(31)^2 \\ N_2^2 &= O(12)^2 + O(22)^2 + O(32)^2 \\ N_3^2 &= O(13)^2 + O(23)^2 + O(33)^2 \end{aligned} \quad (50)$$

From the condition that b, c are real and positive, we have the free parameter a allowed in the range of

$$-\left(\frac{m_e^{-1}m_\mu^{-1}m_\tau}{m_e^{-1} - m_\mu^{-1} + m_\tau^{-1}}\right)^{\frac{1}{2}} < a < 0 \quad (51)$$

or

$$\left(\frac{m_e^{-1}m_\mu^{-1}m_\tau}{m_e^{-1} - m_\mu^{-1} + m_\tau^{-1}}\right)^{\frac{1}{2}} < a < (m_e^{-1}m_\mu^{-1} + m_\mu m_\tau - m_e^{-1}m_\tau^{-1})^{\frac{1}{2}} \quad (52)$$

Now we repeat the previous analysis. The class III with inverted hierarchy are now found to be unacceptable by current experimental data. We present the allowed region for class III with normal mass hierarchy in Fig.3. It is observed that no bound is set on θ_{12} and θ_{13} . However the Dirac CP-violating phase δ is restricted in two regions. We denote them respectively as R1: $0^\circ < \delta < 60^\circ$ ($300^\circ < \delta < 360^\circ$) and R2: $90^\circ < \delta < 150^\circ$ ($210^\circ < \delta < 270^\circ$). Each shows different predictions. In R1, θ_{23} varies in its 3σ range. We obtain a highly suppressed $m_{ee} \simeq 0\text{eV}$ which is beyond the accuracy of future $0\nu\beta\beta$ experiments and implies the underlying cancelation of three neutrino masses in m_{ee} . There also exists the lower bound on the lightest neutrino mass $m_1 > 0.02\text{eV}$. On the other hand, in R2 we find $\theta_{23} < 45^\circ$ which is supported by 2σ experimental constraint. We also obtain $|J_{CP}| > 0.03$. Moreover, the bound on m_{ee} is founded in the range of $0.075\text{eV} \sim 0.2\text{eV}$ and can be potentially detected by future experiment.

IV. COFACTOR ZEROS IN CHARGED LEPTON MATRICES

One reminds the type-I seesaw mechanism as $M_\nu = -M_D M_R M_D^T$. Then the cofactor zeros of M_ν are attributed to the texture zeros in M_D and M_R . Generally, this can be easily realized by Abelian Z_n flavor symmetry[11, 30]. Can the cofactor zeros in M_l arise using the same way? At the tree level, it is obviously impossible. At the loop

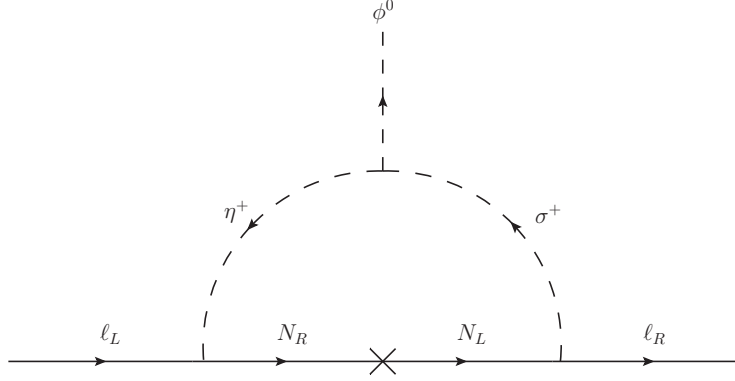


Figure 4: The one-loop diagram for generating radiated charged lepton masses

level, the answer is yes! Here we adopt the model proposed by Ma[21], consisting of the SM extended by adding three Dirac singlet neutral fermion $N_k (k = 1, 2, 3)$, a doublet scalar (η^+, η^0) and a charged singlet σ^+ . In Ma's model, the particles transform under the proper $U(1)_D$ gauge symmetry and A_4 flavor symmetry. Here, we choose the $Z_2^{(A)}$ instead of A_4 flavor symmetry under which N_k , (η^+, η^0) and σ^+ are odd. To forbid the tree level Dirac lepton mass, another $Z_2^{(B)}$ symmetry is imposed such that l_R and σ^+ are odd while others are even. Actually, the flavor symmetry we propose is the same as the one in Ref[31] where the Dirac neutrino mass is generated at one-loop level. The allowed Yukawa interactions are $y_{ij} \bar{N}_{iR} (l_{jL} \eta^+ - \nu_{jL} \eta^0)$ and $h_{ij} \bar{l}_{iR} N_{jL} \sigma^-$. The $Z_2^{(B)}$ is allowed to be softly broken by the trilinear term $\mu(\eta^+ \phi^0 - \eta^0 \phi^+) \sigma^-$ with the SM vacuum expectation $v = \langle \phi^0 \rangle$. The one-loop charged lepton mass is thus generated as shown in Fig. 4, the result being

$$(M_l)_{ij} = \frac{\sin 2\theta}{32\pi^2} \sum_k y_{ik} M_k \left[\frac{m_1^2}{m_1^2 - M_k^2} \ln \left(\frac{m_1^2}{M_k^2} \right) - \frac{m_2^2}{m_2^2 - M_k^2} \ln \left(\frac{m_2^2}{M_k^2} \right) \right] h_{kj}^\dagger \quad (53)$$

The $m_{1,2}$ and θ denote the eigenvalues and the mixing angle of mass squared texture

$$\begin{pmatrix} m_\sigma^2 & \mu v \\ \mu v & m_\eta^2 \end{pmatrix} \quad (54)$$

with

$$m_{1,2}^2 = \frac{1}{2} [m_\eta^2 + m_\sigma^2 \mp \sqrt{(m_\eta^2 - m_\sigma^2)^2 + 4\mu^2 v^2}] \quad (55)$$

and

$$\sin 2\theta = \frac{2\mu^2 v^2}{\sqrt{(m_\eta^2 - m_\sigma^2)^2 + 4\mu^2 v^2}} \quad (56)$$

For $M_k \gg m_{1,2}$, (53) is simplified as

$$(M_l)_{ij} \simeq \frac{\sin 2\theta}{32\pi^2} m_1^2 \sum_k F\left(\frac{m_1^2}{m_2^2}, \frac{M_k^2}{m_1^2}\right) y_{ik} \frac{1}{M_k} h_{kj}^\dagger \quad (57)$$

with

$$F(x, y) \equiv x \ln\left(\frac{y}{x}\right) + \ln y \quad (58)$$

Following the same strategy of Ref.[32], the $F\left(\frac{m_1^2}{m_2^2}, \frac{M_k^2}{m_1^2}\right)$ is treated as a constant at leading order if three M_k are assumed to be nearly degenerated. Then we get

$$M_l \sim m_1 y (M_N)_{diag}^{-1} h^\dagger \quad (59)$$

On the other hand, if we assume $m_\eta \simeq m_\sigma \simeq M_k$ and note $\mu v \ll M_k^2$, then

$$(M_l)_{ij} \simeq \frac{\mu v}{16\pi^2} \sum_k y_{ik} \frac{1}{M_k} h_{kj}^\dagger \sim \mu v y (M_N)_{diag}^{-1} h^\dagger \quad (60)$$

The expression also appears in [33] where the Majorana neutrino mass is generated at one-loop level. From (59) and (60), the charged leptons acquire the radiated masses via the seesaw-like mechanism and masses of heavy Dirac neutral particles N_k play the role of seesaw scale.

Consider now the weak basis where the mass matrix of M_N is not diagonal. It is obvious that, working in the context of the seesaw-like mechanism with a diagonal Dirac matrices y and h , the vanishing cofactors in the charged lepton mass matrix are equivalent to texture zeros in the heavy Dirac fermion mass matrix M_N . As having done in neutrino sector, the texture zeros in y , h , and M_N are easily achieved by introducing extra Z_n flavor symmetries. Form eq. (60), it is clear that the seesaw-like scale M_N is reduced to TeV by the smallness of factor $\mu v/16\pi^2$ originated from softly broken $Z_2^{(B)}$.

V. CONCLUSION AND DISCUSSION

In this work, we have studied the parallel structures with cofactor zeros in lepton mass matrices. These matrices can not be obtained from arbitrary Hermitian texture by making WB transformations. Using the permutation transformation, the 15 possible textures are grouped into 4 classes where the matrices in each class lead to the same physical implications. Among the 4 classes, one of them is not compatible with experimental results and another is equivalent to the texture zero structures explored extensively in previous literature. We focus on the other two classes (class II and class III). Using the new results from the neutrino oscillation and cosmology experiments, a systematic and phenomenological analysis is proposed for each class and mass hierarchy. We have demonstrated that some predictions for the atmosphere mixing angle θ_{23} , the Dirac CP-violating phase δ and the Majorana effective neutrino mass m_{ee} are rather interesting and deserve to be explored in the future experiments. We also demonstrate how the cofactor zeros arise in a seesaw-like model where charged lepton mass are generated at one-loop level. We expect that a cooperation between phenomenological study and the flavor symmetry study will finally help us reveal the structure of leptonic texture.

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